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Rebound and jet formation of a fluid-filled sphere

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This study investigates the impact dynamics of hollow elastic spheres partially filled with fluid. Unlike an empty sphere, the internal fluid mitigates some of the rebound through an impulse driven exchange of energy wherein the fluid forms a jet inside the sphere. Surprisingly, this occurs on the second rebound or when the free surface is initially perturbed. Images gathered through experimentation show that the fluid reacts more quickly to the impact than the sphere, which decouples the two masses (fluid and sphere), imparts energy to the fluid, and removes rebound energy from the sphere. The experimental results are analyzed in terms of acceleration, momentum and an energy method suggesting an optimal fill volume in the neighborhood of 30%. While the characteristics of the fluid (i.e., density, viscosity, etc.) affect the fluid motion (i.e., type and size of jet formation), the rebound characteristics remain similar for a given fluid volume independent of fluid type. Implications of this work are a potential use of similar passive damping systems in sports technology and marine engineering.
 [http://dx.doi.org/10.1063/1.4771985]

I. INTRODUCTION

The sloshing phenomenon has prompted significant research on the design of large tanks that store valuable fluids. The effects of sloshing are not limited to the movement of fluid but are also a powerful disturbance to the stability of marine and land-based containers.[1–4] Although an external motion initiates the sloshing motion, the moving fluid subsequently affects the physical motion of the container in return.[5] Sloshing, resulting from sudden changes in direction or external oscillatory disturbances, must be considered for almost any moving vehicle or structure containing liquid with a free surface (the surface of fluid exposed to air).[6] In fact, the powerful forces induced by sloshing fluid can be harnessed in a way to stabilize tall buildings by the use of a tuned liquid damper.[7,8] These dampers are placed in the base or upper floors to counteract the forces caused by winds or seismic activity.

The sporting industry also uses fluids to stabilize the performance of street hockey balls. Street hockey balls[9] utilize sloshing-induced damping to remove the vertical rebound after a ball is hit, keeping it on the ground. These balls are partially filled with a viscous fluid which acts to reduce the rebound response of the sphere. This study investigates this phenomenon further, unraveling the motions of a spherical container partially filled with various liquids. To simulate the extreme circumstances under which sloshing may occur, partially filled spheres were dropped from rest. The impact of the first bounce results in the formation of a cavity-like deformation in the free surface of the fluid. At the moment of the second impact, this cavity collapses as the fluid mass contained in the walls of this cavity experience an instantaneous change in acceleration and a powerful jet forms, shown in Fig. 1, and the rebound of the sphere is dramatically reduced. The visible events of the rebounding motion of this sphere can be observed in Fig. 2.

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FIG. 1. Jet formation within a partially fluid-filled sphere. m = 173 g, dia = 8 cm.

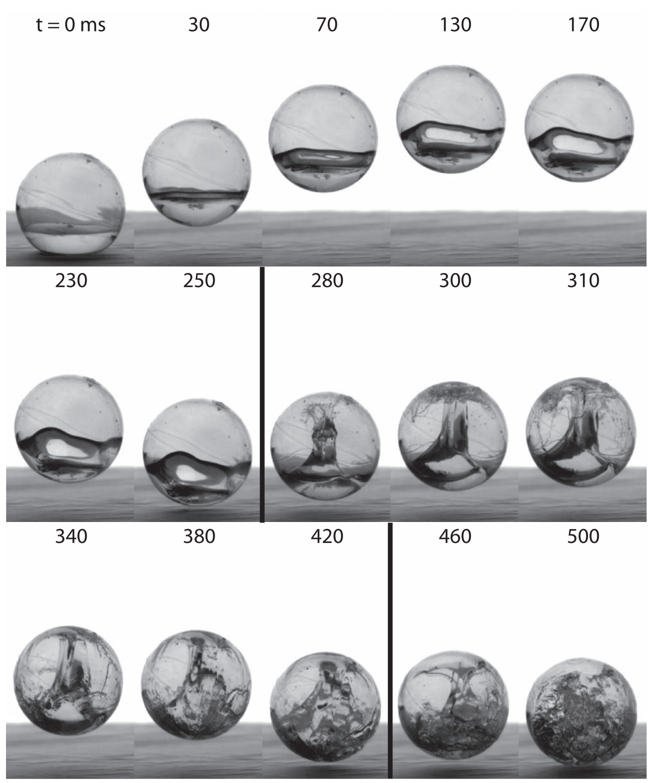


FIG. 2. Progression of a urethane sphere filled with 20% water dropped from a height of 10 cm. (m = 207 g, dia = 8 cm.) The time in milliseconds is marked above each image. The moment of greatest sphere deformation upon the first impact is marked as t = 0 ms. The apex of the first rebound is marked at t = 130 ms. Dark lines indicate where the second and third impacts occur. The apex of the second rebound occurs at t = 340 ms. After the first impact, the fluid climbs up the sides of the sphere and forms a large cavity in the center. On the second impact, this cavity collapses forming
Q1 a large jet in the center. On the third impact, the flow becomes completely disorganized (Video 1) (enhanced online). [URL: http://dx.doi.org/10.1063/1.4771985.1].

TABLE I. Summary of independent parameters varied in the experiment.

Sphere type	Diameter [cm]	Weight [g]	Drop height [cm]	Fluid density and viscosity [$\mathrm{kg\,m^3}$, $\mathrm{N\,s\,m^{-2}}$]	Fill volume [%]
Skyball® (TPU)	8	125	10, 20, 30, 60	Water [998, 8.9e^{-4}] Iso. Alcohol [786, 1.92e^{-3}] Glycerin[1261, 9.5e^{-1}]	0, 10, 20 30, 40, 50 60, 70, 80 90, 100
Acrylic	8	50	10, 20, 30, 60	Water [998, 8.9e^{-4}]	0, 30, 70, 100

According to the experimental data presented in this study, it appears that the majority of the sphere's rebound is removed at the second impact and resulting rebound. However, this momentum transfer can occur after the first impact if the fluid interface is initially perturbed prior to release of the sphere (see Sec. III D). The momentum transfer and rebound mitigation was found to be optimal when the sphere was filled to 30% of the interior volume, which is roughly equal to the mass of the empty sphere, independent of the physical properties of the fluids used.

Many researchers have established and validated the foundational work of Worthington.[10] For example, Antkowiak *et al.*[11] demonstrated that jet formation is dependent upon the dynamic collapse of a meniscus-like deformation in the free surface. Gekle *et al.*[12,13] showed the local phenomenon of jet formation and growth due to the drastic changes in direction and momentum that occur following the collapse of cavities created by plunging disks and other objects through the free surface. Thoroddsen and Shen[14] and Lohse *et al.*[15] noticed similar dynamics of jet formation in granular materials. Bergmann[16] summarized many of the common examples of jet formation while noting that the size and curvature of the cavity as well as the speed of that cavity's collapse led to varying jet formations.

The studies performed by Antkowiak *et al.*[11] and Gekle *et al.*[12,13] begin to describe the jet formation as a deposit of the kinetic energy contained in the collapsing walls of the cavity. Rognebakke and Faltinsen[2] noted that the momentum acquired by a tank filled with sloshing fluid was reduced (albeit on a very small scale) by the formation of fluid jets as the sloshing fluid interacted with its container. These observations led to the primary hypothesis of this study, that the suppression of the rebound of a partially filled sphere is caused by an exchange of energy from the bouncing sphere to the internal motion of the fluid. The primary experimental objective was to determine, through the variation of physical parameters (summarized in Table I), an optimum fluid level or type within the sphere for maximum rebound mitigation.

II. EXPERIMENTAL METHODS

The interaction between internal sloshing and rebound suppression is manifest by fluid motion inside the spheres as they fall and rebound. Transparent spheres were used to highlight this motion and to improve understanding of what is occurring inside while measuring the position outside. High-speed imaging was used to capture the position and visualize the fluid motion.

Spheres were dropped over a range of heights, to measure the rebound response with regard to initial conditions, using the empty sphere as a baseline for comparison. These spheres were filled with fluids of different density and viscosity in increments of 10% of the interior fill volume to determine the effect of the added fluid on the rebound of the partially filled spheres (shown in Table I). Two types of spheres were used to investigate the influence of elasticity: a Skyball® and a rigid acrylic sphere. The Skyball® is a thin walled (3–5 mm) thermal plastic urethane (TPU) sphere originally pressurized with oxygen and helium (0.65–0.8 kg/cm^3). The internal pressure was removed and fluid was added or removed by using a hypodermic syringe. Removing the pressure reduced the diameter of the sphere to those listed in Table I. The acrylic spheres were made up of two thin walled (1.75 mm) halves glued together and filled via a small hole in the top covered with tape before use.

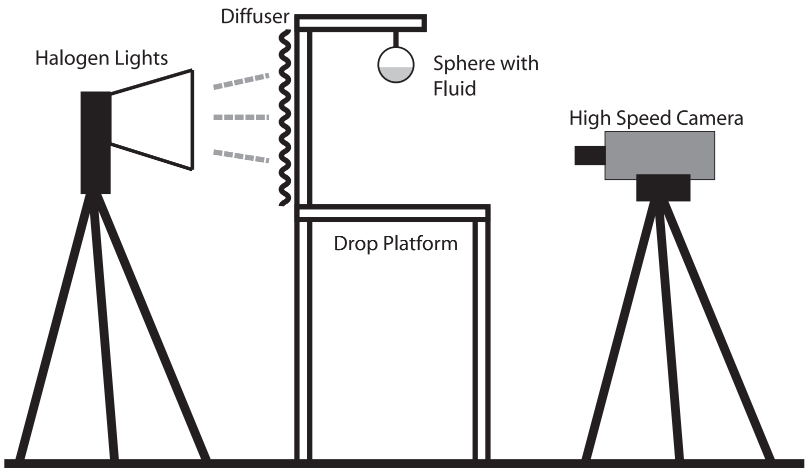


FIG. 3. The experimental setup used to observe the dynamics of a partially fluid filled sphere.

The spheres were suspended above a drop platform made of an acrylic plate ($61 \times 61 \times 5$ cm^3) supported by a custom-built frame (Fig. 3). The height of the spheres was varied by raising and lowering a suction cup attached to the spheres. Back lighting was provided by several halogen lights, placed behind a diffusion screen. Images were captured by a Photron SA3 camera at 1500 frames per second (fps) with a shutter speed of 1/6000 s.

The sphere positions were tracked using a combination of methods. First, a 2D normalized cross-correlation algorithm compared each image to a template of a portion of the sphere similar to the methods in Truscott.[17] In cases where this method failed, a sequential difference between images was used to find the leading edge.[18] Although both of these methods can contribute to large errors when calculating velocity and/or acceleration, the data for position estimates was used exclusively which have relatively low error, $\pm$ 3 pixels (i.e., $\pm$ 0.997 mm on average depending on the image set).

III. EXPERIMENTAL RESULTS

A. Fluid motion

When a partially filled sphere impacts the ground, the collision imparts considerable motion to the enclosed fluid. Depending on the fluid properties, the surface geometry, and the initial drop height, the fluid motion produces cavities, jets, swirl, and chaotic flow. Water, isopropyl alcohol, and glycerin were used to study density and viscous effects. The rebound characteristics of each fluid type were compared with that of an empty sphere (see Fig. 4) to determine the effect the fluid had on the rebound suppression. The effort to quantify this effect is discussed and demonstrated in Sec. IV.

Unlike water (Fig. 2, Figs. 4(c) and 4(d)), the alcohol (Fig. 4(b)) immediately forms a radial jet after the first impact that travels up through the center of the sphere, converging as it rises, enclosing a cavity of air inside itself. When the top of the radial jet closes, two jets are formed, one directed upwards and the other down into the cavity. Surprisingly, there is no noticeable effect on the initial rebound height as the sphere rebounds to approximately 63% $\pm$2% of its initial height, similar to the empty sphere in Fig. 4(a). The radial jet formation remains until the second impact, upon which it collapses downward and the flow becomes chaotic and the rebound is suppressed by 15%–75%, depending on the fluid volume.

In the water-filled sphere (Fig. 4(c), $V < 50\%$), after the initial impact the water travels up the sides of the sphere forming a smooth surface with a depression in the middle. This fluid motion has no apparent affect on the first rebound and the sphere travels to approximately 6 1% $\pm$2% of the drop height, similar to the empty sphere (Fig. 4(a)). On the second bounce, the cavity collapses toward

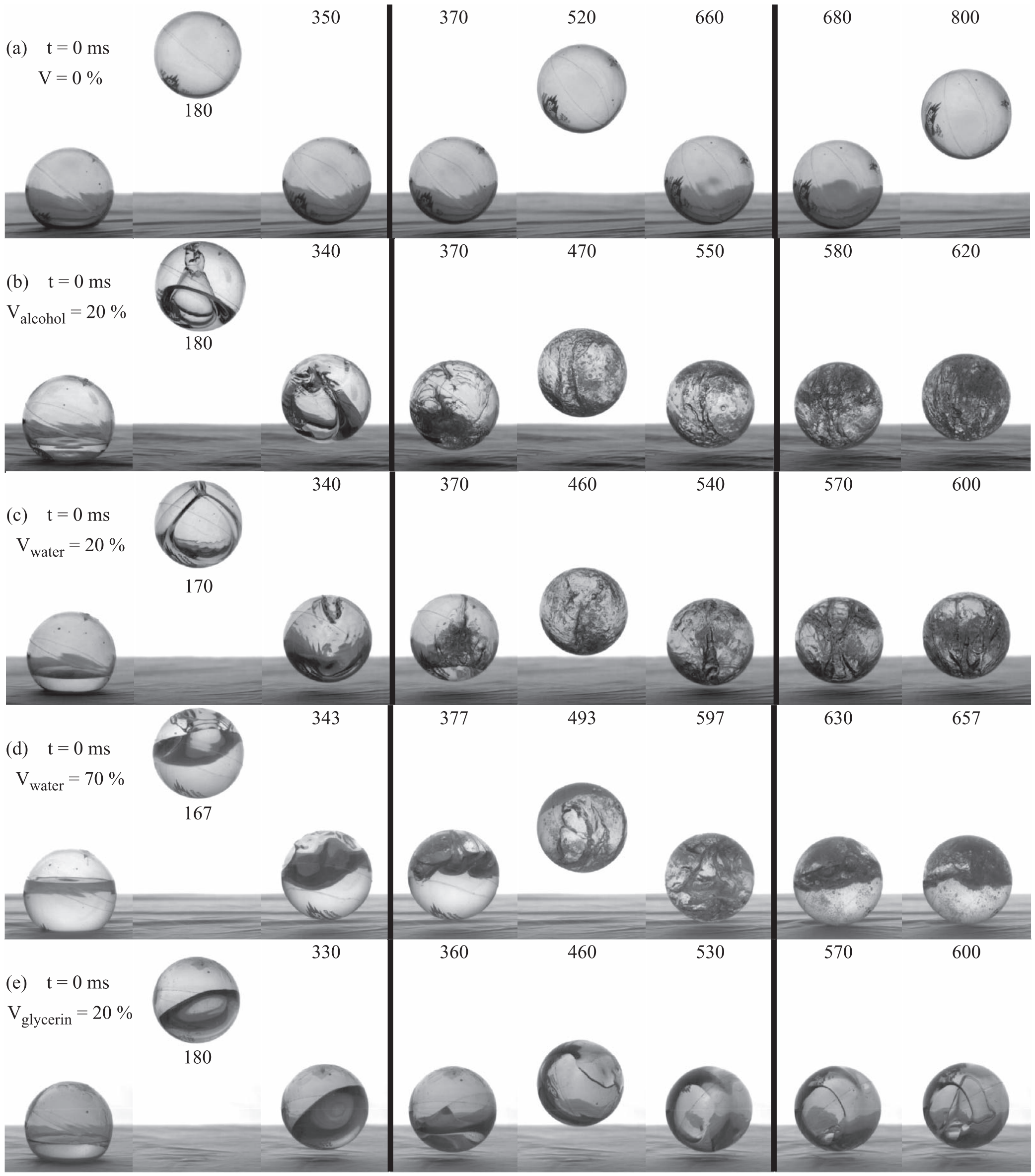
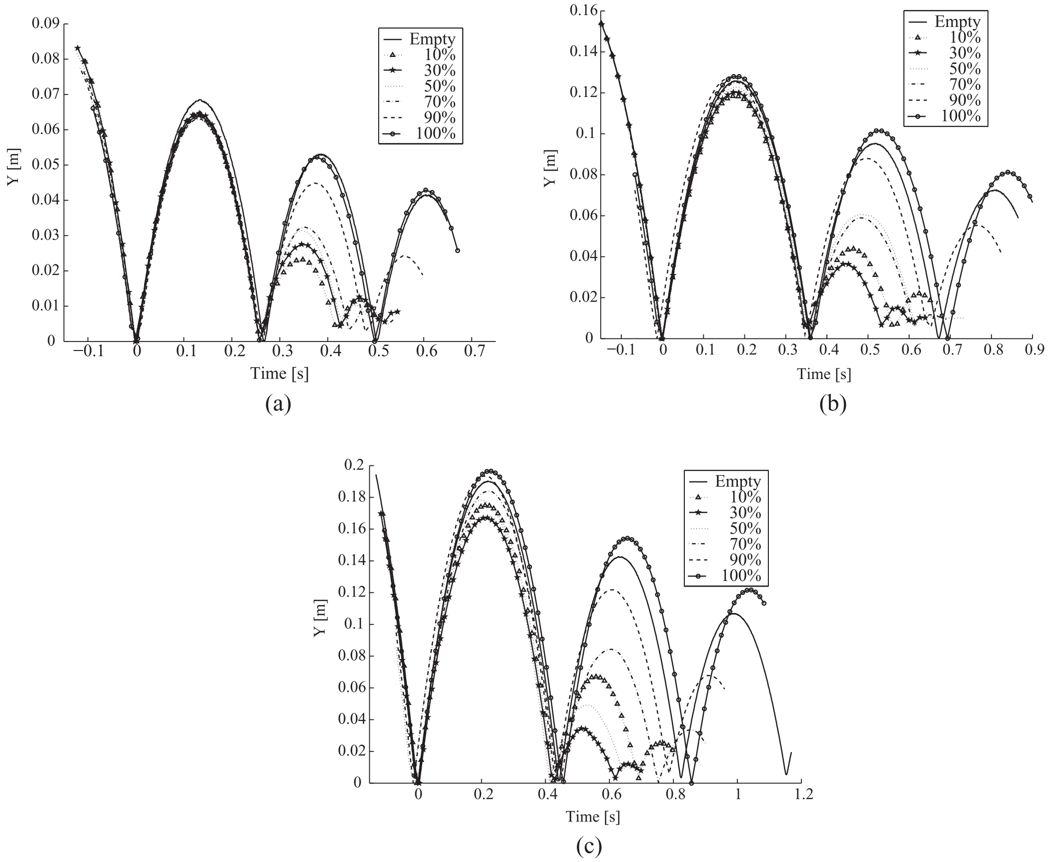


FIG. 4. Comparison of fluid motion with different volumes and physical properties within a urethane sphere. All sequences included are dropped from 20 cm. The moment of greatest sphere deformation upon the first impact is marked as t = 0 ms. Dark bars indicate the second and third impacts. (a) Sequence of an empty sphere (Video 2) m = 125 g. (b) Sequence of a sphere 20% filled with isopropyl alcohol (Video 3) m = 196 g. (c) Sequence of a sphere 20% filled with water (Video 4) m = 207 g. (d) Sequence of a sphere 70% filled with water (Video 5) m = 425 g. (e) Sequence of a sphere 20% filled with glycerin (Video 6) m = 249 g. All spheres shown have a diameter of 8 cm (enhanced online). [URL: http://dx.doi.org/10.1063/1.4771985.2] [URL: http://dx.doi.org/10.1063/1.4771985.3] [URL: http://dx.doi.org/10.1063/1.4771985.4] [URL: http://dx.doi.org/10.1063/1.4771985.5] [URL: http://dx.doi.org/10.1063/1.4771985.6].

its base forming a vigorous jet that impacts the top interior of the sphere. When the fluid volume is greater than 50% (Fig. 4(d)) the fluid forms into a single, wide jet after the first impact. (This phenomenon is discussed in greater detail in Sec. III D.) After the second impact and corresponding jet formation, fluid motion inside the sphere becomes chaotic until the sphere eventually comes to rest. After the second impact of all water filled trials the rebound of the sphere is greatly suppressed, reaching between 20%–70% of the previous apex. The large variance in the second rebound appears to depend on the amount of water in the sphere as well as the initial drop height.



Q2 FIG. 5. Rebound trajectories vs time for several cases of a sphere partially filled with water: empty sphere, 10%, 30%, 50%, 70%, 90% and 100% filled sphere. The 30% filled sphere experiences the greatest rebound suppression of any case dropped from 20 cm (compare to Fig. 4). (a) Trials dropped from 10 cm. (b) Trials dropped from 20 cm. (c) Trials dropped from 30 cm.

The glycerin filled sphere also experiences a greatly reduced fluid motion during rebound (Fig. 4(e)). After the first impact the glycerin begins to travel up one side of the sphere and a shallow depression is formed in the center. The initial rebound height is not affected by the presence of the glycerin for reasons other than the greater mass of the fluid. Immediately after the second impact, a small jet forms off center as the cavity collapses. The fluid then circles up and around the other side, coating the wall with a layer of glycerin. The sphere motion is suppressed to approximately the same degree as water and alcohol.

B. Rebound suppression

Of all the parameters varied, the fluid volume inside the sphere had the greatest effect on rebound suppression. For the urethane spheres an empty sphere rebounds to approximately 85% of the previous height. As shown in Fig. 5, the sphere rebound is greatly reduced as small amounts of fluid are added. The first rebound is relatively unaffected by the presence of fluid. However, the second rebound is dependent on the fluid volume, achieving a minimum height at 30% filled and a maximum at 100% (i.e., reaching nearly the same height as an empty sphere, see Fig. 5).

The initial drop height has very minor effects on the rebound characteristics of the spheres. As the drop height increases, the effects due to viscosity and density are diminished. Also, as the initial drop height changes, the fluid volume required for maximum suppression varies slightly in a neighborhood of the previously optimal 30%, except for drop heights less than or equal to 10 cm.

The rebound trajectories of spheres filled with water, alcohol, and glycerin to fill volumes of 10%–40% versus time are shown in Fig. 6. Maximum suppression occurs in all three cases

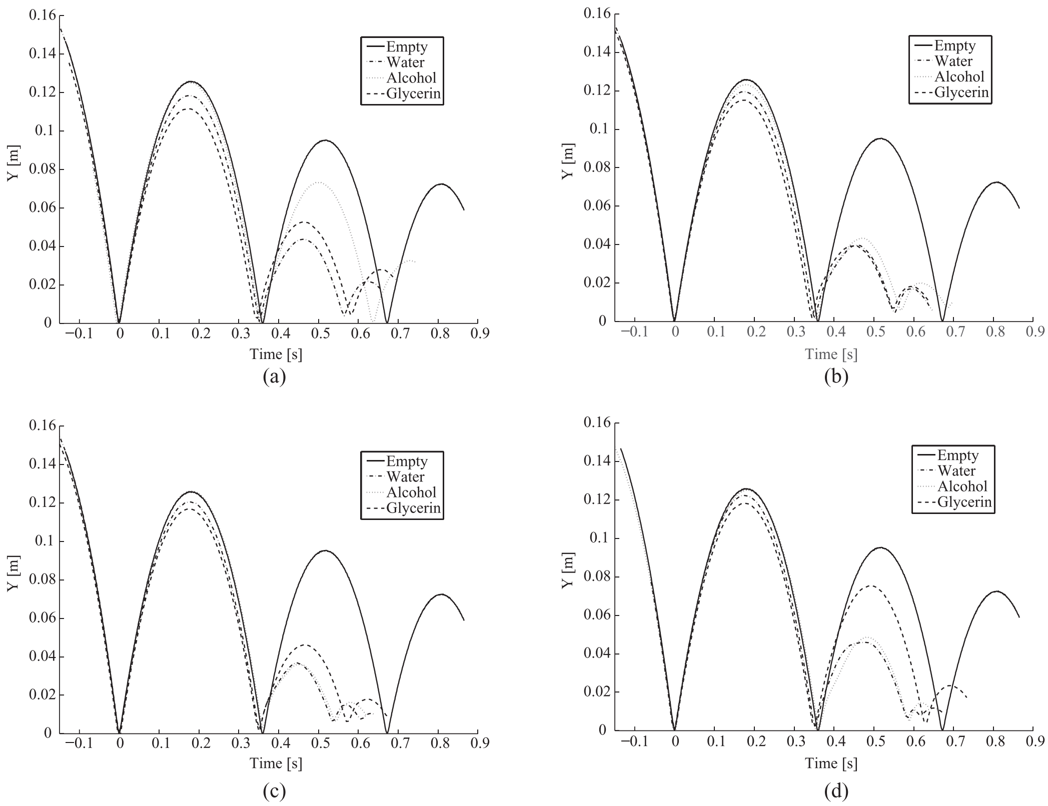


FIG. 6. Rebound trajectories vs time for trials of water, isopropyl alcohol, and glycerin filled to 10%–40% of interior volume in comparison with that of an empty sphere. Trials are dropped from 20 cm (compare to Fig. 4). (a) 10% filled. (b) 20% filled. (c) 30% filled. (d) 40% filled.

when the fluid volume is approximately 30% of the total interior volume of the sphere. Given the similarity between the responses of different fluids across a wide range of volumes tested (Fig. 6), fluid properties do not seem to play a role in rebound suppression unless the initial drop height is extremely low ($\leq$10 cm). It is supposed that the dynamics of rebound suppression of a partially fluid-filled sphere is relatively independent of viscosity.

C. Elasticity

Street hockey balls are made of a firm, hard polymer that deforms less than the TPU shown in Fig. 4. To address the role that elasticity plays in rebound suppression, acrylic spheres were partially filled with water and tested using the same method as outlined in Sec. II (see Table I). Water was the only fluid type used with these trials because the role of viscosity was not apparent in the rebound suppression of the urethane spheres (the case of a completely filled sphere was too heavy to be dropped from 30 cm without breaking).

An image sequence of the progression of 30% and 70% filled trials with an acrylic sphere can be seen in relation to an empty acrylic sphere in Fig. 7. The inelasticity of the acrylic reduces the time that the sphere is in contact with the impacting surface. This limited contact time and relatively low material deformation causes the fluid to react more rapidly.

In the 30% trials (Fig. 7(b)), the fluid response mirrors the 30% filled urethane spheres of Fig. 4(c) only accentuated due to the rapid fluid response from the impact. A meniscus-like cavity forms shortly after the first impact, the walls of which almost cover the entire interior surface. After the second impact, jet formation is visible as the cavity collapses. This flow pattern quickly transitions to chaotic motion. Similar to the urethane cases, rebound suppression is greatest for the 30% filled cases of the acrylic sphere as seen in Fig. 8.

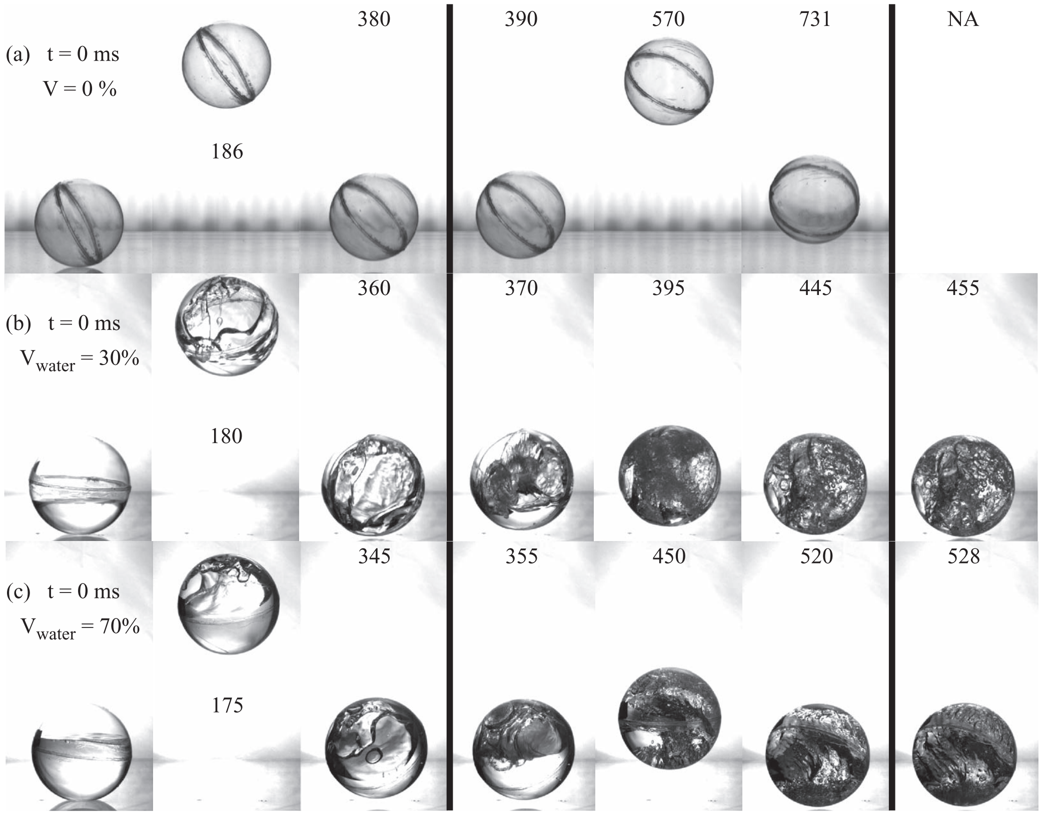


FIG. 7. Comparison of fluid motion with different volumes and physical properties within an acrylic sphere. All sequences included are dropped from 20 cm. The moment of first impact is marked as t = 0 ms. Dark bars indicate the second and third impacts. (a) Sequence of an empty acrylic sphere (Video 7). m = 50 g. (b) Sequence of an acrylic sphere 30% filled with water (Video 8). m = 192 g. (c) Sequence of an acrylic sphere 70% filled with water (Video 9). m = 387 g. All spheres shown have a diameter of 8 cm (enhanced online). [URL: http://dx.doi.org/10.1063/1.4771985.7] [URL: http://dx.doi.org/10.1063/1.4771985.8] [URL: http://dx.doi.org/10.1063/1.4771985.9].

In the 70% trials (Fig. 7(c)), the fluid response again mirrors that of a similarly filled urethane sphere in an accentuated manner. Following the first impact, the fluid forms into a wide jet originating from the center of the free surface. As the sphere rebounds, the jet impacts the top of the sphere before the apex of the first rebound. Following the second impact, the flow becomes chaotic, while the sphere rebounds to a reduced height as can be seen in Fig. 8.

A key difference between the trials of an acrylic sphere and those of an urethane sphere is that the completely filled acrylic sphere does not rebound as high as expected. In the urethane trials, the trajectory of a completely filled sphere was nearly identical to that of an empty one, whereas the completely filled acrylic has a reduced rebound height (Fig. 8).

D. Points of interest

Dropping a sphere partially filled with fluid from rest, provides an understanding of the coupling and later decoupling of the fluid mass with its container under sloshing motions. If one is to truly investigate the dynamics of the rebound of a street hockey ball and how fluid acts to mitigate that rebound, it is necessary to disturb the initial free surface of the fluid. Experiments involving an initially disturbed free surface were briefly investigated, utilizing three different methods to disturb the quiescent conditions (explained above): creating a vortex in the fluid, a randomly disturbed free surface and rolling off of an inclined plane.

If a vortex is created in the fluid by hand-rotation before release, a large central jet and a reduced rebound height occurs after the initial impact (Fig. 9) similar to the second rebound of the cases in

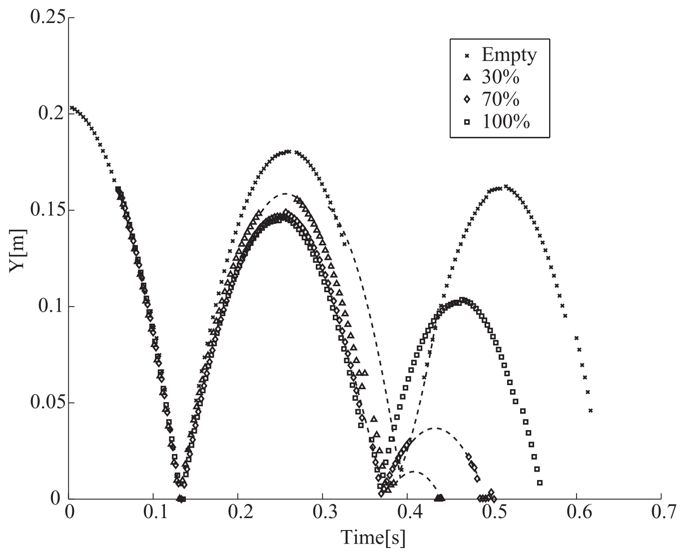


FIG. 8. Rebound trajectories vs time for trials of partially filled acrylic spheres with water: empty sphere, 30% filled sphere, 70% filled sphere, and 100% filled sphere. Trials are dropped from 20 cm (compare to Fig. 7). Again we see that an interior fill of 30% mitigates the rebound best from that drop height. Inconsistencies in tracking the motion of the sphere due to overexposure were approximated with a quadratic fit, hence the gaps in the data replaced by dotted lines.

Fig. 4. This confirms that a disturbed free surface is an important parameter for energy transfer from the sphere to the fluid. Note that the size of the jet formation in Fig. 9 is somewhat reduced when compared to the spheres of Fig. 4 because much of the energy transfer changes the direction of the vortical motion in addition to the jet.

As more fluid was added to both the acrylic and urethane spheres, they elongated slightly, while suspended due to the greater mass acting on the material. After the spheres were released, the material oscillated. These oscillations imparted a resonance to the fluid, resulting in a gently disturbed free surface by the time of the first impact (Fig. 10). The disturbed surface gives way to several small jets after the first impact where under normal, undisturbed circumstances this formation would not arise until the second impact.

Another method to observe the rebound of the sphere was to have the sphere roll off of a shallow incline where impact occurred in the viewing area of the positioned cameras. This scenario was observed in cases where the fluid was initially undisturbed and when a vortex was formed in the fluid. Similar to the suspended sphere which included a vortex, the rolling sphere with rotating fluid had a greatly reduced first rebound in relation to the rolling sphere whose fluid had not been subjected to rotation (Fig. 11(a)). The fluid response of the cases where the fluid is initially undisturbed are

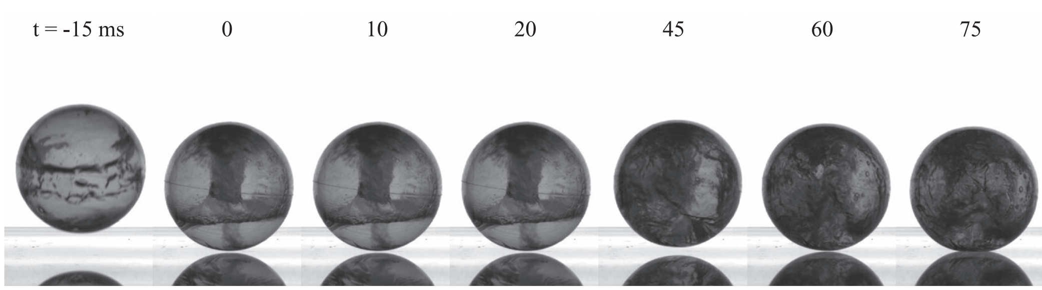


FIG. 9. Images of the first rebound for a 30% filled urethane sphere with an initially disturbed free surface (vortex) dropped from 20 cm above the ground. The vortex collapses during the first rebound (t = 25 ms) forming a large jet that mitigates the rebound height. m = 250 g, dia = 8 cm (Video 10) (enhanced online). [URL: http://dx.doi.org/10.1063/1.4771985.10].

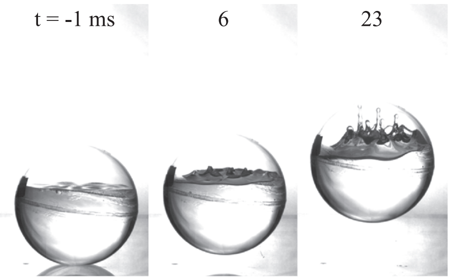


FIG. 10. Images of a 70% filled acrylic sphere that was initially elongated before release, resulting in small oscillations afterward (same set as Fig. 7(c)). A jet is formed as each of the small dimpled disturbances collapse after impact. m = 387 g, dia = 8 cm (Video 11) (enhanced online). [URL: http://dx.doi.org/10.1063/1.4771985.11].

similar to those explained in Sec III; a cavity-like deformation forms after the first impact which collapses into a jet upon the second impact and rebound is mitigated (Fig. 11(b)).

Control was difficult to replicate in all of these situations, so extensive experimentation was not performed. The limited sample set provides a qualitative understanding that initially imparting a rotation to the fluid serves to mitigate rebound. This verifies the conclusions that a motion imparted to the free surface, naturally or by experimentally augmenting the initial conditions, serves to mitigate the rebound of a partially filled sphere.

IV. ANALYSIS

This section explores the jet formation, momentum exchange, and kinetic energy losses needed to describe the transfer of energy from the external motion of the sphere to the internal flow of the fluid. It is clear that there is some exchange of momentum with regard to the experimental data and results described in Sec. III. The dynamics of a partially fluid filled sphere depend on the deformation of the free surface into a cavity and the resultant collapse of that cavity. Antkowiak *et al.*[11] and Gekle *et al.*[12] explained that jet formation is dependent on the transfer of kinetic energy from the collapsing walls of the cavity converging into the vigorous jet. Viscosity's role is disregarded in a general sense with regard to the data shown and discussed in Sec. III B. Here, we conclude that the main damping of the ball motion is caused by a decoupling of the sphere mass and fluid mass rather than the viscous dissipation within the sphere and that the maximum dissipation of energy occurs when the sphere is 30% full.

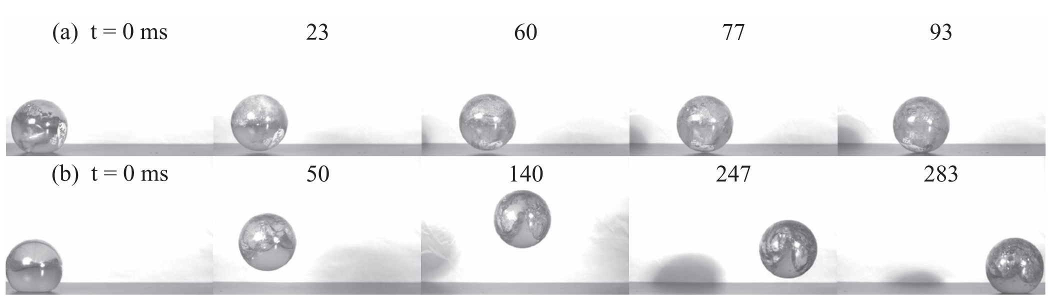


FIG. 11. Images of the first rebound for two 60% filled urethane spheres rolling off of a 30% inclined plane (not shown) from 10 cm above the ground. m = 377 g, dia = 8 cm. (a) Released with an initially rotating vortical-like flow (Video 12). The vortex collapses during the first rebound (t = 23 ms) forming a large jet that mitigates the rebound height. (b) Released after the flow came to rest yielding a much larger rebound (Video 13) (enhanced online). [URL: http://dx.doi.org/10.1063/1.4771985.12] [URL: http://dx.doi.org/10.1063/1.4771985.13].

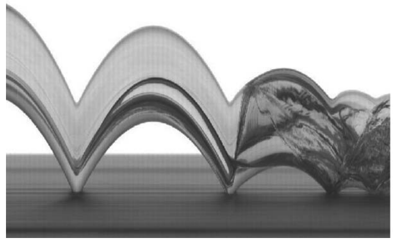


FIG. 12. This image is formed by combining a single pixel line from the center of the sphere from each image in the set of Fig. 2, where the left side is t = −120 ms and the right is t = 536 ms.

A. Jet formation

The jet is a result of the collapse of the meniscus-like cavity formed after the first rebound. Both the meniscus-like cavity and jet are the result of accelerations in the fluid, therefore, determination of these accelerations and their directions seems appropriate. We can approximate them by measuring the motion of the meniscus and jet at one location inside the sphere. To do this, we analyze a line of pixels along the center of the sphere as shown in Fig. 12 similar to the technique used by Gilet and Bush.[19] This removes some of the motion of the meniscus and jet outside the pixel-line, but yields a reasonable approximation. The results of this analysis are shown in Fig. 13 where the position, velocity, and acceleration of the sphere and cavity/jet (relative to the sphere position) are plotted on top of one another. In the case of the fluid, measurements after the jet impacts the top of the sphere are impossible due to the chaotic nature of the motion, thus we impose a zero value after full jet formation in Fig. 13 (Y_j).

The position of the meniscus is flat and has relatively no motion before the first impact in Fig. 13 (Y_j, t = 0 ms) and begins to form a cavity after impact, visible in the increasing position of the meniscus until the sphere reaches its apex (between t = 0–130 ms). As the sphere begins to fall

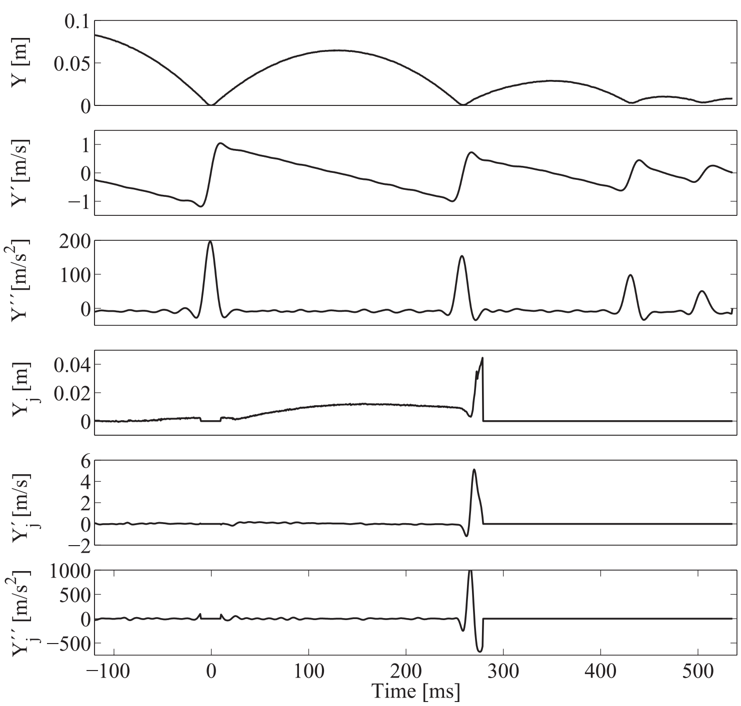


FIG. 13. The position, velocity, and acceleration of both the sphere (Y relative to ground) and interior fluid (Y_j relative to the sphere). Position, velocity, and acceleration of the jet are forced to zero after jet formation (t > 280 ms).

toward the second impact the meniscus appears to settle somewhat and stop moving up the sides of the sphere until impact with the ground where the meniscus collapses very quickly and a large jet is formed (t = 258–270 ms). From the acceleration of the jet it is apparent that the downward motion of the collapsing meniscus creates a negative acceleration on the order of 200 m/s^2 (t = 258 ms), whereas the jet moves upward creating a positive acceleration of the fluid on the order of 1000 m/s^2 (t = 267 ms). While there is some error in the method applied here it is clear that the acceleration of the jet exceeds the acceleration of the collapsing event and that the acceleration of the fluid is important in the cavity/jet formation. These accelerations indicate a large transfer of momentum from the motion of the sphere to the motion of the fluid.

B. Momentum exchange

Since the system appears to be independent of increasingly viscous fluids an explanation of where the damping occurs is warranted. Although we cannot carefully measure the fluid motion of each mass particle inside the sphere, we can estimate velocity of the initial jet and make assumptions about the mass affected to show that a decoupling of the sphere-fluid system is the main factor in the momentum transfer. We propose a simple physical model where two elastic spheres represent the fluid-solid system where the empty elastic sphere is represented by a larger sphere of mass M and the fluid as a smaller sphere of mass m. Furthermore, the mass m is positioned above M such that the centers of the two spheres are perfectly aligned and both spheres are released from rest at the same time (Fig. 14(a)). Sphere M reaches the ground first and reverses direction resulting in a collision with m. The collision results in a loss of momentum for M and a reversal of direction and increased momentum for m.

For the sake of simplicity, we view the collisions of M with the ground and between m as one-dimensional and elastic. That is, momentum and kinetic energy are conserved. The differences in momentum of the two masses prior to the collision contribute to their motion after colliding. When the masses of the two spheres and their initial velocities are known, one can solve for the masses' exit velocities with the following relations (u_m and u_M are the entry velocities, here $u_m = u_M$),

$$v_m = \frac{u_m(m - M) + 2Mu_M}{m + M}, \qquad v_M = \frac{u_M(M - m) + 2mu_m}{m + M}. \tag{1}$$

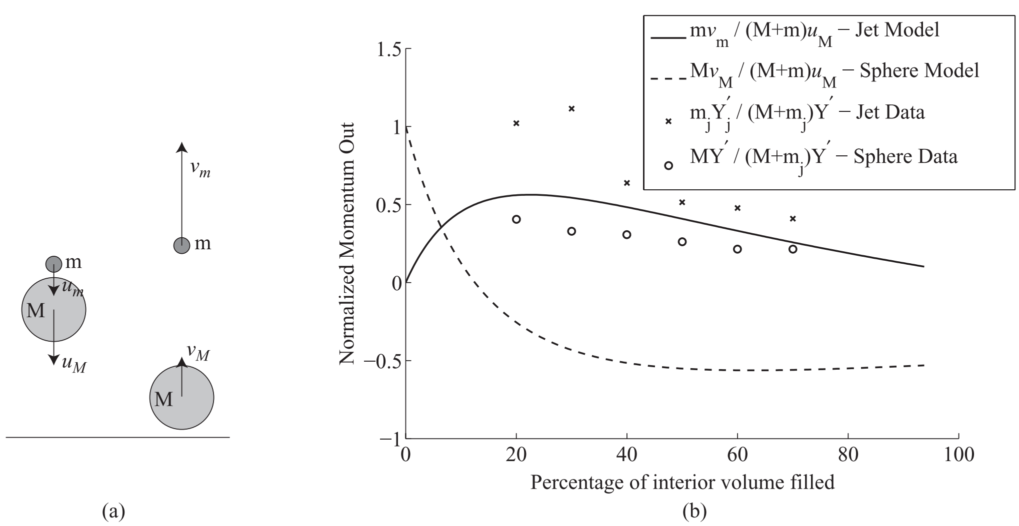


FIG. 14. (a) A physical model of the rebound reduction of a partially filled sphere where two spheres collide after one impacts the ground altering their momentum. (b) The momentum of the sphere and jet after the second impact normalized by the combined momentum of the sphere and jet before impact for both the model and experimental data.

A comparison between the momentum of the sphere and jet normalized by the total incoming momentum for the second rebound is presented in Fig. 14(b). Both data extracted from a water filled urethane sphere dropped from 10 cm and the theoretical analog based on Eqs. (1) are presented. As m increases from 0% to 100% filled the momentum of M decreases for both theory and data. The rebound of M is minimized when M has negative momentum after impact with m. If this were the case in the sphere-fluid system, then the maximum height that the sphere reaches would be constant at volumes ≥30%. However, we do not see this in the rebound heights shown in Fig. 5 nor negative momentum values in the sphere data of Fig. 14(b). Instead, the rebound of the sphere increases even when the mass of the fluid exceeds that of the sphere (≥30% filled).

In the case of the jet, the model predicts that 50% or more of the momentum is transferred to the fluid for the 25% filled sphere. Similarly, the data indicate that more than 100% of the momentum transfers to the fluid for the 30% filled case. Clearly this is impossible, and occurs in our data set because we apply the maximum jet velocity at a single location to the entire fluid volume. From the high-speed image data, it is evident that not all of the fluid is moving at the maximum jet velocity, and debatable whether the entire fluid volume is decoupled from the sphere. However, the data do indicate trending similar to the model and show a decreasing momentum exchange as the volume of the sphere is increased beyond 30%. Though the quantitative comparison between theory and experiment are inaccurate, a qualitative statement arises: A majority of the damping motion is caused by the decoupling of the two masses (fluid and sphere) wherein an exchange of momentum from a rebounding sphere to a chaotically moving fluid significantly reduces the rebound height.

C. Energy loss formulation

Where investigating momentum failed in that measuring fluid velocities was grossly overestimated, calculating a loss of energy from the rebounding sphere to the fluid is facilitated more easily by simple and accurate measurements of rebound height. The mass of an empty sphere (m_{sph}) and the mass of the fluid (m_{fl}) are considered to be separate but coupled. This coupling is defined when the body of m_{fl} is in contact with the walls of the sphere. It is observed that the motion of one directly influences the other, including the moment of jet formation following the second impact. The total energy (*TE*) of the system is given by the first instance where the energy is wholly potential.

The important steps of this system are the initial drop, the first impact and rebound, the apex of the first rebound, the second impact and rebound (resulting in jet formation), and finally the apex of the second rebound as shown in Fig. 15.

The system operates on the reserve of *TE* created by the sphere held at an initial height. This reserve is depleted as energy is lost in the dynamic interaction between the sphere and the impacting surface, as described by the coefficient of restitution of the sphere material. Much of the remaining

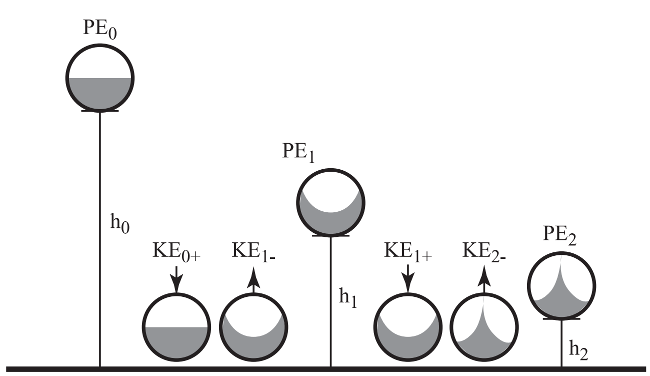


FIG. 15. A free body diagram of a partially fluid-filled sphere. At the initial position, (PE_0), m_{tot} is suspended above a flat, rigid surface at the initial height h_0. As the sphere rises from the first impact (KE_{1-}), a parabolic cavity forms. At the apex of the first rebound, (PE_1), the cavity is fully formed and the energy of the system is once again stored in the potential of m_{tot} at h_1. The kinetic energy of m_{tot} approaching the second impact is transferred to the collapse of the cavity and kinetic energy contained in m_{fl}. The result of this energy transfer is the formation of the jet (KE_{2-}) and reduced rebound height, h_2.

energy of the combined mass m_{tot} after two impacts is lost through the decoupling of m_{sph} and m_{fl}. The reduction of energy across an arbitrary impact can then be resolved in both dynamic losses $[dyn_n]$(due to elasticity) and fluid losses $[fl_n]$, which can be expressed as a difference of potential energies.

The energy transferred to the fluid can be expressed by the change in potential energy from rebound to rebound as potential energy is proportional to height. The most dramatic change in rebound heights, as observed in Sec. III, occurs between the first and second rebounds as the fluid cavity collapses and a jet is formed. Consequently, it is not necessary to continue the investigation of energy loss beyond the second rebound. So our system is represented by

$$TE = PE_n + fl_1 + dyn_1 + fl_2 + dyn_2.$$

Therefore, the sphere's energy losses to the fluid following the first impact can be expressed as:

$$TE = PE_n + dyn_1 + fl_1,$$

$$fl_1 = m_{tot}g(h_0 - h_1) - m_{sph}g(h_0 - h_{1_0}). \tag{2}$$

The sphere's energy losses to the fluid after the second impact can then be found using Eq. (1),

$$TE = PE_n + fl_1 + dyn_1 + fl_2 + dyn_2,$$

$$fl_2 = m_{tot}g(h_1 - h_2) - m_{sph}g(h_{2_0} - h_{1_0}). \tag{3}$$

It can be seen from Eqs. (2) and (3) that the loss of momentum to the fluid at the first and second rebounds depends on the change of potential energy from the preceding rebound (or in the case of the first rebound, from the initial drop height), h_{1_0} and h_{2_0} represent the first and second rebounds of an empty sphere. The preceding rebound height demonstrates the maximum amount of energy that is available for the sphere rebound.

D. Energy loss results

The method described above extracts the dynamics of a partially filled sphere and can be used to observe the effect of different physical parameters in reducing the rebound. To approximate the actual transfer of energy between rebounds, the method described in Sec. IV C was applied to the experimental data presented in Sec. III. Figure 16 shows that a majority of the energy is lost to the fluid during the second rebound. The loss of energy to the fluid after the first impact is minimal in relation to the amount of energy that is provided to that impact, and the loss to the fluid after the second impact is significantly higher in terms of the energy available. As more fluid is put into the sphere, the energy lost to the fluid on the first impact increases. The added mass of the increasing fluid volume provides a greater potential for energy reduction upon the initial impact. The energy that is lost to the fluid after the second impact varies, depending on drop height and interior fill volumes, with a maximum occurring within a neighborhood of a 30% fill volume.

The results of the energy transfer as described for the partially filled sphere are independent of the fluid viscosity. As can be noted between the two plots featured in Fig. 16, the differences in the energy transferred due to a change in viscosity are slight. The greatest transfer of energy is after the second impact regardless of the fluid viscosity. Again, the maximum energy loss after the second impact occurs in a neighborhood of a 30% fill volume regardless of viscosity, which is when the mass of the fluid is approximately equal to the mass of the sphere.

When the *TE* of the system exceeds the potential for suppression in the fluid, the second rebound of the sphere is greater than those cases where the majority of the *TE* is lost to the fluid. The fluid volume also has a limitation to how much energy it can take from the system. Increasing the total energy of the system will, at some point, overcome the potential for energy reduction by the fluid. Simply adding fluid to the spheres in these cases does not counter the greater initial energy. There appears to be a threshold of fill volume that optimally suppresses the second rebound of the sphere.

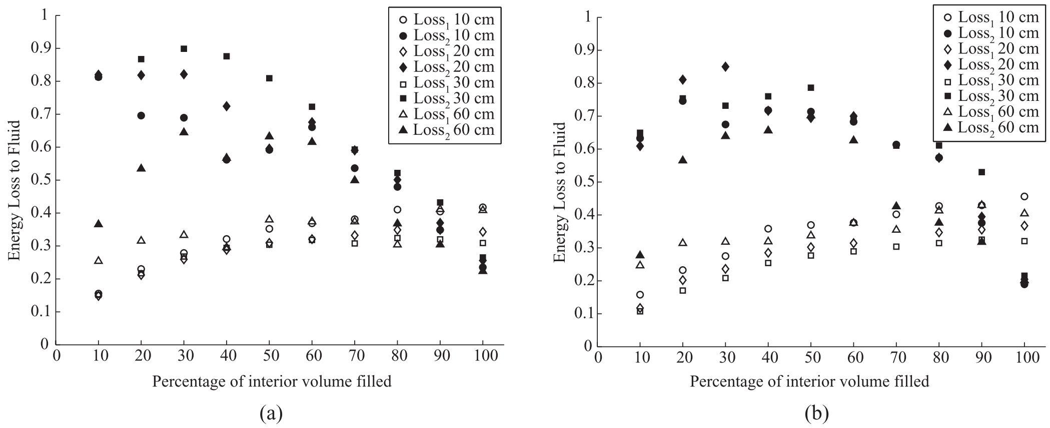


FIG. 16. Non-dimensional energy loss vs percentage of interior volume filled for trials of (a) urethane spheres partially filled with water and (b) urethane spheres partially filled with isopropyl alcohol. Empty markers denote the energy loss to the fluid occurring after the first impact, while the filled markers represent the loss after the second impact. The results are scaled by the amount of energy available to the sphere before first and second impact, Mgh_0 and Mgh_1, respectively.

Once this threshold is passed, the second rebound of the sphere is higher with each increment of added fluid until the completely filled sphere nearly mirrors the trajectory of the completely empty sphere, this fact is highlighted in the reduced transfer of energy after the second impact of the completely filled sphere.

Calculating the energy losses from rebound to rebound provides an understanding of the optimal fill volume for rebound suppression. Without explanation, Quinn[9] states (in the patent of his street hockey ball) that the street hockey balls are to be filled to a level less than half of the interior volume of the ball. As can be seen in Fig. 16, his observations are verified. In fact, across all variations of fluid viscosity and drop height, the energy removed from the system when the sphere is filled with less than 50% of the interior volume is significantly greater than those trials with a greater fill volume (with ≈30% optimal).

Understanding the definition of how m_{fl} and m_{sph} are coupled helps to explain why an interior fill volume greater than 50% does not further reduce the rebound of the partially filled sphere after the second impact. When the volume of fluid inside the sphere increases from 0% to 50%, the area of the fluid free surface increases. With greater surface area, there is more room for the fluid jet to form and be decoupled from the combined mass M. There is more space available in these cases for the fluid to move away from the boundary of the sphere. As more fluid is added to the sphere (50%–100%), the free surface area decreases and the available space for the decoupling of the fluid mass from M is restricted.

Extensive trials were performed for spheres dropped from 10 to 30 cm with energy loss calculations agreeing relatively well after normalizing by the amount of energy available prior to the sphere's impact. Viewing the output of the energy loss from the runs dropped from 20 cm and 30 cm led to an assumption that the data converged toward some upper limit of energy removed by the fluid as the total energy of the system was increased. To determine whether this assumption was valid, trials were run dropping the sphere from 60 cm (see Fig. 16). The results show a reduction in the energy taken out of the system by the fluid after the second bounce. This invalidates the assumption that a greater initial energy would provide a closer fit to the general trend observed with the earlier trials, although better agreement was observed among trials using alcohol.

The energy transfer results of filling an acrylic sphere with water to volumes of 30%, 70%, and 100% exhibit similar responses as the urethane spheres in terms of the percentage of energy that is transferred to the fluid after the first and second impacts (Fig. 17). The initial fluid response and rebound characteristics of the acrylic sphere differs from that of the urethane sphere, as discussed in Sec. III C, but the overall behavior of energy loss to the fluid is in agreement. That is, the greater energy loss is found in the cases that have fill volumes less than 50%, and the losses accrued after the

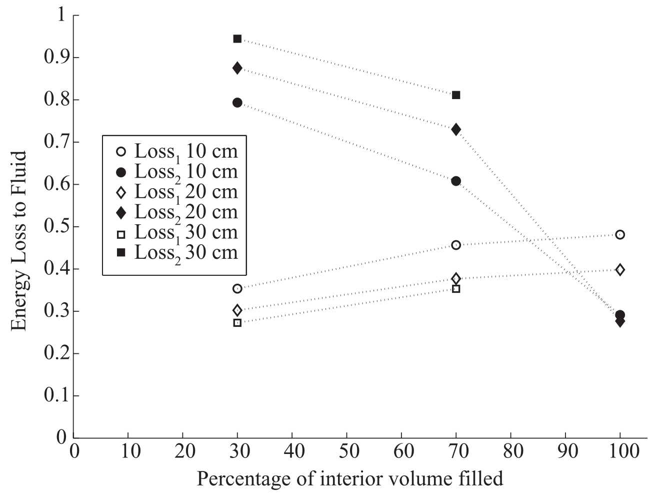


FIG. 17. Non-dimensional energy loss vs percentage of interior volume filled for acrylic spheres that are partially filled with water. Scaling and marker representation are the same as in Fig. 16. Dotted lines are included to assist in observing the behavior of the system as more fluid is added.

first impact are greater than those of the second impact in the cases where the sphere is completely filled. An event explained by the emergence of small jets after the first impact due to small oscillatory disturbances in the free surface (Sec. III D).

V. CONCLUSIONS

When a partially fluid filled sphere impacts with the ground, a jet is formed when a deformation in the free surface collapses. The jet acts to mitigate a large portion of the rebound energy in a decoupling of the momentum from the sphere to the fluid. For initially quiescent flows, this occurs after the second rebound since the first rebound acts to form a large cavity-like deformation. This phenomenon is utilized by the Quinn[9] street hockey ball which claims that the desired fill volume is less than half of the interior volume. We confirm and add that the ideal fill volume varies between 10% and 40% depending on the initial drop height of the sphere. However, the most consistent fill volume for rebound suppression among fluid and sphere material types was 30%.

Within a partially filled sphere, the fluid flow is only constrained by the sphere itself. This provides an inherent understanding of the relationship between fluid and sphere masses that governs the rebound mitigation and momentum transfer. Analyzing the energy lost to the fluid leads to several conclusions. The greatest amount of energy lost to the fluid inside the sphere occurs in a neighborhood of a 30% fill volume. The moment of the second impact, resulting in cavity collapse and jet formation, contributes most directly to this transfer, which is independent of viscosity, despite the changes in the jet formation structure observed in Fig. 4. Increasing the total energy of the system by raising the initial drop height does not necessarily lead to greater energy transfer. A change in material properties, from urethane to acrylic, does not affect the mechanism of energy transfer although the specific fluid responses differ. Foremost, the mechanism of energy transfer from the sphere to the fluid is a decoupling of the two masses and is only effective in the presence of a free surface.

This passive removal of energy from a moving component is a potentially cost effective method for stabilizing systems. In particular, for systems engaged in the interaction with fluids—where the motion of an object and the fluid are coupled—this method of stabilization could benefit the system's designed purposes. Obviously, sloshing and jet formation on a large scale can damage and incapacitate mechanisms and containers. [3] However, harnessing sloshing as a passive dampener is potentially advantageous on a small scale. The application of this study to larger systems could be achieved by combining several small dampeners to effectively mitigate larger motions and is the subject of future work.

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